Coincident Bit Counting—A New Criterion for Image Registration

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Abstract—Image registration is a very important operation in multichannel image applications to correct for the frame-to-frame displacement that occurs during the image acquisition process. In order to register a pair of images, an image registration algorithm often employs some similarity criterion between the image functions, testing different displacement vectors to find an extremum of this similarity measure. A good similarity criterion should be robust with respect to the diverse noise environments encountered in multichannel image applications. Misregistration often leads to invalid results in later image processing stages.

In this paper, a new similarity measure based on the number of coincident bits in multichannel images is presented. The similarity criterion incorporated in the image registration algorithm uses a coincident bit counting (CBC) method to obtain the number of matching bits between the frames of interest. The CBC method not only performs favorably compared with traditional techniques, but also renders simpler implementation in conventional computing machines. An image registration algorithm which incorporates the CBC criterion is proposed to determine the translational motion among sequences of images. The analysis of the errors caused by noise, misregistration, and a combination of these two is also included. Some experimental studies using low-contrast coronary images from a digital angiographic sequence have been performed. The results compare favorably with those obtained by using other nonparametric methods. Applications for this algorithm includes digital angiography and mammography.

I. INTRODUCTION

MULTICHANNEL images, whether acquired at different temporal instants, different spatial locations or in different frequency bands, can provide valuable information in many medical applications, such as digital angiography and mammography. In digital subtraction angiography, due to the continuous motion of the heart and coronary arterial tree, a straightforward subtraction of images in an angiographic sequence will not render useful results. For early detection of nodules locations, mammographic images obtained from right and left breasts have to be compared. However, due to the different shape, size and contrast of right and left radiographs, a preprocessing which extracts the relations among radiographs taken from different angles has to be performed before further comparisons or operations can be made. In order to extract useful information from the acquired multichannel images, a registration procedure is often applied to relate the scenes in different frames. This registration step is usually accomplished by optimizing some similarity measure between the frames of interest subject to the registration parameters.

In conventional image registration, one of three kinds of measurement is usually chosen: correlation coefficient, correlation function and the sum of the absolute values of differences [1], [2]. In order to improve the performance of the similarity measure and reduce the operational requirements, a preprocessing step is often performed. However, these methods only work correctly when the features of interest to be compared are nearly identical. Misregistration often occurs when significant differences exist between two images. For example, lead materials set on the scintigraphic image of a radioactive liver phantom have been used to demonstrate the invalidity of the above similarity criteria in the registration of dissimilar objects [3]. These major changes often have special clinical significance, and are the focuses in the study of multichannel images, as in the case of pictures taken before and after a bone graft plastic surgery. Human discretion is needed to judge the correctness of the registration result, which prevents the use of these techniques in an automated image registration system.

A new class of similarity measures based on nonparametric statistical considerations has been proposed to improve registration performance [3]–[6]. These methods either take the pixel value difference of two image frames or artificially introduce a sign change pattern by periodically adding a fixed value to the pixel value difference. The number of sign changes is then computed for various displacement vectors, and the displacement vector corresponding to the maximum number of sign changes is chosen as the displacement estimate. These nonparametric methods have been shown to work well both in similar or dissimilar situations. However, these methods assume a symmetric distribution for the noise after the subtraction operation. In the next section, we will show this assumption is not always valid by considering a signal-dependent Poisson noise model and discussing its effect on the degradation of the performance of these nonparametric statistical methods.

The reason that these nonparametric approaches outperform the conventional ones lies in the fact that the value of their similarity measure does not take the specific pixel values into account. Each pixel constitutes the same amount of weighting in the similarity measure irrespective of its pixel value. In light of this observation, we propose a similarity measure based on the number of matching bits between the corresponding pixels of adjacent frames. This chosen similarity measure not only
performs well relative to the above algorithms, but does so without assuming a priori knowledge regarding the nature of noise.

The image registration criterion introduced in this paper is called the coincident bit counting (CBC) method. The CBC method compares the number of coincident bits between the corresponding pixels in two different frames for a fixed amount of displacement. The displacement estimate is obtained when the number of matching bits reaches a maximum. The number of bits to be matched can be dynamically adjusted according to the noise level. This flexible setting permits a higher resolution when more information about the nature of noise is known. Noise, relative motion, and a combination of these two may cause pixel values to deviate from the original ones, which results in an estimation error. Errors caused separately by noise and motion are analyzed first, and then a combinatorial method is employed to evaluate the compound effects. Experimental studies using images obtained synthetically and clinically show promising results. Since the CBC method does not assume any specific noise distribution, it is more robust than the nonparametric ones, especially in low-intensity imaging conditions where the noise model follows a signal-dependent Poisson counting process.

In what follows, we will first briefly discuss the methodology of the nonparametric statistical methods and a signal-dependent Poisson noise model where the performance of these methods will suffer. We then introduce the proposed CBC criterion and an image registration algorithm incorporating it. Analysis of the CBC method affected by noise and misregistration is presented next. Later, experimental results are described and comparisons between different criteria are made. Finally, some perspectives for future work are given.

II. NONPARAMETRIC STATISTICAL METHODS

The nonparametric statistical methods, which will be discussed in Section II-A, take advantage of the assumption of a symmetric noise distribution after a subtraction operation to determine the matching location. These methods work well as long as the noise model is continuous and symmetric, and can be applied to registration of both similar and dissimilar objects. In Section II-B, a signal-dependent Poisson noise model, which is representative of images obtained at low intensities, will be examined. Also, we will demonstrate that the performance of these statistical methods degrades rapidly when the noise model deviates from the continuous and symmetric assumptions, as in the aforementioned signal-dependent Poisson noise case. While nonparametric statistical methods are robust in the sense of registering similar or dissimilar objects, they are very sensitive to the underlying noise model. It is the purpose of this section to show that deviation from the continuous and symmetrical assumptions causes these nonparametric statistical methods to become unstable and more likely to lead to misregistration.

A. SSC and DSC

A new class of similarity measures based on nonparametric statistical considerations has been proposed to improve registration performance over conventional methods like correlation function and correlation coefficient. When the noise level in the images is greater than the digitization precision, the stochastic sign change (SSC) criterion is applied by first taking the pixel value difference of two image frames \( F_1 \) and \( F_2 \):

\[
D_{\text{ssc}}(i,j) = F_1(i,j) - F_2(i,j). \tag{1}
\]

We call \( D_{\text{ssc}} \) the subtraction image of \( F_1 \) and \( F_2 \) in a sequence of images. If \( F_1 \) and \( F_2 \) are perfectly aligned, the subtraction image \( D_{\text{ssc}} \) is just the superposition of noises from \( F_1 \) and \( F_2 \). If this superpositional noise is symmetrically distributed about zero, the statistically best registration coefficients correspond to the maximum number of sign changes in \( D_{\text{ssc}} \). Based on the assumption of additive Gaussian noise, a 95% confidence interval can be derived for the SSC criterion [3]

\[
(S_{\text{low}}, S_{\text{high}}) = \left( \frac{N}{2} - 1 - 1.96\sqrt{\frac{N}{4} - 2}  - 1 + 1.96\sqrt{\frac{N}{4}} \right)
\]

where \( S_{\text{low}} \) and \( S_{\text{high}} \) are lower and upper bounds for the confidence interval.

When the noise levels are low, the subtraction image \( D_{\text{ssc}} \) in (1) becomes null. A deterministic sign change (DSC) criterion is then applied. In order to produce a fixed pattern of sign changes, a fixed value \( q \), determined by the noise variance, is periodically added to or subtracted from the subtraction image \( D_{\text{ssc}} \):

\[
D_{\text{dsc}}(i,j) = D_{\text{ssc}}(i,j) + (-1)^{i+j}q. \tag{3}
\]

The number of sign changes is then computed, and the selected registration coefficients correspond to the maximum number of sign changes in \( D_{\text{dsc}} \) [6].

When the noise levels are high, the precision of the digitized image will suffer due to either re-quantization or truncation. Thus SSC will not perform as well as predicted. In the case of DSC, in order to introduce a fixed pattern of sign change, the optimum \( q \) value chosen depends on the noise variance, which is not necessarily known a priori. Moreover, the performance of these nonparametric statistical methods depends strongly on the assumption that the residual noise in the subtraction image is continuously and symmetrically distributed, which contributes to the value of the probability of sign change of 0.5 in the SSC case and underlies the derivation of the confidence interval in the Gaussian noise case mentioned above. We will discuss this point further in the remaining parts of this section.

B. Signal-Dependent Poisson Noise

In the process of acquiring a sequence of digitized medical images, noise is added at each stage of generation and capture of the image. A predominant source is quantum noise, caused by the limited and Poisson-distributed number of incident X-ray photons [7], [8]. As described by Kuan et al. [9], a signal-dependent Poisson noise model can be developed from the expression

\[
P(g(i,j)|f(i,j)) = \frac{(\lambda f(i,j))^g(i,j)e^{-\lambda f(i,j)}}{g(i,j)!} \tag{4}
\]
where \( f(i,j) \) is the true pixel value, \( g(i,j) \) is the noisy version of \( f(i,j) \) and \( \lambda \) is a proportionality factor which translates the photon counts into the appropriate range of pixel values. The value of \( \lambda \) is constant for each frame, but may change from one frame to another.

The normalized Poisson observation can be represented in terms of signal and signal-dependent additive noise. The noise part can be obtained by subtracting the original pixel value \( f(i,j) \) from the noisy version \( g(i,j)/\lambda \).

\[
   n(i,j) = \frac{g(i,j)}{\lambda} - f(i,j).
\]

Consider two frames of an image sequence corrupted by the signal-dependent Poisson noise described above. In applying the SSC method after subtracting these two images, the noise part contained in the subtraction image is no longer symmetrically distributed about zero. Moreover, the probability of the residual noise at the origin is not zero since the Poisson process is a discrete model. We can prove the above statement by evaluating the skew of the residual noise by subtracting the noise parts \( n_1 \) and \( n_2 \) belonging to the two frames,

\[
   E((n_1 - n_2)^3) = f \times \frac{(\lambda_2^2 - \lambda_1^2)}{\lambda_1^2 \lambda_2^2}.
\]

Since \( \lambda_1 \) and \( \lambda_2 \), the proportionality factors for the two frames, are not necessarily the same, we conclude the skew of the residual noise after subtraction of two images is not equal to zero. This finding leads us to the conclusion that the noise part is not symmetric [lo], i.e.,

\[
   p_+ = \text{Prob}(\text{noise} > 0) \neq \text{Prob}(\text{noise} < 0) = p_-.
\]

Furthermore, we see that the probability of sign change is lower than 0.5 in the SSC case:

\[
   p_{sc} = \text{Prob(\text{sign change})} = 2p_+p_- < 0.5.
\]

This finding is clearly inconsistent with the noise model that the nonparametric statistical methods assume. Next, we will show how their performance degrades due to the deviation from the symmetrical noise model.

The confidence level of the interval \((S_{\text{low}}, S_{\text{high}})\), as described in (2), is a function of \( p_{sc} \) which can be formulated as follows:

\[
   \text{C.L.}(S_{\text{low}}, S_{\text{high}}) = \sum_{n=0}^{S_{\text{high}}} p_{sc}^{n}(1 - p_{sc})^{N-1-n}.
\]

In Fig. 1, we show two curves of the confidence level versus different \( p_{sc} \) values corresponding to image size 100 and 1000, respectively. As we can see, the value of the confidence level falls rapidly as \( p_{sc} \) deviates from 0.5. The tendency for this confidence level to fall below 0.95, the value predicted by SSC, gets even worse as the image size increases up to 256 \( \times \) 256 or even 512 \( \times \) 512. This result supports the statement that the performance of these nonparametric statistical methods are very sensitive to the assumption of a symmetric noise model and may not necessarily render satisfactory results in some application environments.

III. CBC CRITERION

The similarity criterion that we introduce in this section uses a coincident bit counting (CBC) method to determine the displacement vector from one frame to another. This CBC method will not be affected by the specific pixel values compared, only the number of matching bits. Thus it is equally applicable to the registration of similar or dissimilar objects. Also, it does not assume any \textit{a priori} knowledge about noise distribution, so the performance is robust in applying it in diverse noise environments.

The CBC method has the desirable property that when more information regarding the nature of image and noise is available, its performance can be enhanced by flexibly adjusting the number of bits compared. Lower order bits tend to be contaminated by noise, and thus are not suitable for reliable comparison. Higher order bits tend to be locally uniform among neighboring pixels, especially in the case of low-contrast images, and thus will render lower resolution. By excluding error-prone and locally uniform bits during bit comparison, a steeper search surface can be formed. The steepness of the search surface under various noise models can be used as a basis in comparing the performance of different similarity criteria. An analysis of the CBC method without assuming any specific noise distribution, which leads to the formulation of the relative steepness, is presented in the next section.

In Fig. 1, we show two curves of the confidence level versus different \( p_{sc} \) values corresponding to image size 100 and 1000, respectively. As we can see, the value of the confidence level falls rapidly as \( p_{sc} \) deviates from 0.5. The tendency for this confidence level to fall below 0.95, the value predicted by SSC, gets even worse as the image size increases up to 256 \( \times \) 256 or even 512 \( \times \) 512. This result supports the statement that the performance of these nonparametric statistical methods are very sensitive to the assumption of a symmetric noise model and may not necessarily render satisfactory results in some application environments.

Consider two frames \( P \) and \( Q \) in a sequence of multichannel images. The CBC criterion performs bit-by-bit logical comparison of the corresponding pixel values \( P(i,j) \) and \( Q(i,j) \).
where 1 indicates a bit match and 0 a mismatch. If \( P(i, j) \) and \( Q(i, j) \) contain two noisy versions of the same scene, this bit comparison will be affected by the level of noise only. However, if \( P(i, j) \) and \( Q(i, j) \) represent the values of two misplaced pixels, the bit comparison will be affected by both the noise level and the values of misaligned pixel values. Counting the number of matching bits across the area compared, we can use it as a similarity measure to indicate the registration of two frames corresponding to certain displacement coefficients.

A neural implementable image registration algorithm that incorporates this CBC criterion to determine the two-dimensional displacement vector \((\Delta x, \Delta y)\) from one frame to another can be formulated as follows: find \((\Delta x, \Delta y)\) such that

\[
F(\Delta x, \Delta y) = \frac{P(x, y) \odot Q[(x + \Delta x) \mod (L+2), (y + \Delta y) \mod (L+2)]}{(L - |\Delta x|)(L - |\Delta y|)},
\]

is maximum

\[
P(x_1, y_1) \odot Q(x_2, y_2) = \begin{cases} 
  P(x_1, y_1) \odot Q(x_2, y_2) & \text{if } \max(l, l + \Delta x) \leq x_1, x_2 \leq \min(L + l, L + l + \Delta x) \\
  \max(l, l + \Delta y) \leq y_1, y_2 \leq \min(L + l, L + l + \Delta y) & \text{otherwise}
\end{cases}
\]

Each frame contains \( L \) by \( L \) pixels, and \( L \) is the maximum displacement in either \( X \) or \( Y \) direction between frames of interest. The operator \( \times \) is just a normal binary multiplication operator. The operator \( \odot \) is an Exclusive-NOR operator which gives one for matching bits and zero otherwise.

The purposes for defining these operators are to mask out the nonoverlapping area using the \( \times \) operator while at the same time match the pixel value by using the \( \odot \) operator. The numerator term in \( F(\Delta x, \Delta y) \) serves as a bit counter that counts the number of matching bits in the current overlapping area, which effectively is a measure of similarity, while the denominator term provides a normalization factor that tracks the number of pixels contained in the current overlapping area. This image registration algorithm searches for the vector \((\Delta x, \Delta y)\) that has the highest number of matching bits per pixel.

The SSC method described in Section II depends solely on the statistical properties of the noise model. The assumed noise process has to be continuous and symmetric about zero after the subtraction of two perfectly aligned images. However, this assumption may not always be true, as in the case of images acquired in low-intensity conditions where the noise process can be modeled as signal-dependent Poisson noise. On the other hand, the CBC method does not assume any specific noise distribution. Only the magnitude of the noise will affect the performance of CBC method, not the symmetry of its distribution. The number of comparison bits used in the CBC method can be adjusted depending on image contrast and noise level.

IV. ANALYSIS OF CBC METHOD

A fairly complicated analytic result regarding the value of the CBC method has been obtained and its derivation will be briefly described in this section. This derivation does not assume any specific noise distribution and can be followed by substituting the actual noise probability density function.

In applying the CBC method to estimate frame-to-frame motion, usually a reference frame is arbitrarily chosen first and the remaining frame, termed the registration frame, is oriented spatially to match the reference frame. A bit mismatch may occur due to two factors: noise corruption and misregistration. For the reference frame, since it is kept stationary to provide a reference for registration, only noise contamination will cause the discrepancy of bit values from the original one. However, an erroneous bit value in the registration frame may be caused not only by noise but also by misregistering the pixel position. The noise effect will be analyzed first to give the probability of bit errors in the reference frame. The misregistration effect will then be considered and combined with the results of the noise effect to give the probability of bit errors in the registration frame. Finally, the probability of bit mismatch is derived by relating the probabilities of error bits in both frames.

1) Bit Error Due to Noise: Let \( B = (b_1, \ldots, b_n) \) represent a vector of the bit values of an 8-bit deep pixel. Assume \( n \) bits among \( B \) are corrupted by additive noise, \( 1 \leq n \leq 8 \), i.e., \((b_{n+1}, \ldots, b_8) = b_c\) where \( b_c \) is the set of error bits in \( B \), which are not necessarily contiguous, and \( b = b_u \) is the set of complement values of \( b_c \), i.e., the corresponding correct values before noise contamination takes effect. In what follows, we will define four interval notations first to simplify our derivation.

Let

\[
(i_n, \ldots, i_1) = (2^{2n} + \ldots + 2^{i_1} - 0.5),
\]

\[
(2^n + \ldots + 2^{i_1} + 0.5),
\]

\[
(i_n, \ldots, -i_1, \ldots, i_1) = (2^{i_1} + \ldots + (-2^n))
\]

\[
+ \ldots + 2^{i_1} - 0.5,
\]

\[
(2^n + \ldots + (-2^n))
\]

\[
+ \ldots + 2^{i_1} + 0.5),
\]

\[
(i_n, \ldots, i_1) = \{2^n + \ldots + 2^{i_1} - 0.5, \infty\},
\]

\[
(i_n, \ldots, i_1) = \{\infty, 2^n + \ldots + 2^{i_1} + 0.5\}.
\]

The above notations of intervals are indications of quantization levels. Any pixel value falling into the range specified on the right-hand side of the equations will be quantized as the corresponding left-hand side binary digits.

Let \( \epsilon \) represent the value of additive noise. For the noise-corrupted \( n \) bits, there are \( 2^n \) possible combination patterns before and after the addition of noise:
The other 2^n - 2 possible combinations are
\[ 0 < b = (0, \cdots, 0, 1) \mapsto b_e = (1, \cdots, 1, 0) \]
\[ b = (0, \cdots, 1, 0) \mapsto b_e = (1, \cdots, 1, 0) \]
\[ b = (1, \cdots, 1) \mapsto b_e = (0, \cdots, 0, 1) \]
\[ \epsilon \in (-i_m, \cdots, -i_1) w/\text{prob.} \left( \frac{1}{2} \right)^n \]

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\[ b = (1, \cdots, 1) \mapsto b_e = (0, \cdots, 0, 1) \]
\[ \epsilon \in (-i_m, \cdots, -i_2, -i_1) w/\text{prob.} \left( \frac{1}{2} \right)^n \]

The probability that \((b_{i_n}, \cdots, b_{i_1})\) are corrupted by noise can now be calculated as
\[ P_{a}(i_n, \cdots, i_1) = \left( \frac{1}{2} \right)^n \int_{x_n, \cdots, x_1} n(t) dt + \left( \frac{1}{2} \right)^{n-1} \int_{x_n, \cdots, x_1} n(t) dt \]
\[ + \cdots + \frac{1}{2} \int_{x_n, \cdots, x_1} n(t) dt \]
\[ \sum_{S_m} \sum_{S_i} \frac{1}{2} \int_{x_n, \cdots, x_1} n(t) dt \]

where \(n(t)\) is the noise probability density function and \(S_i, S_m \in \{+1\} \).

2) Bit Error Due to Misregistration: After deriving the single-frame error probability caused by noise, we proceed to analyze the misalignment errors caused by relative movement from one frame to the next. A vector of bit-retaining probabilities \((p_{b_0}, \cdots, p_{b_1})\) is introduced to indicate that the \(n\)th bit of the pixel in some spatial location retains its value in successive frames with probability \(p_{b_n}\). This bit retaining probability vector can be obtained statistically from model images and serve as a priori knowledge in analyzing misalignment effects. Based on this assumption, we can derive the probability of bit error due to movement:

\[ \begin{align*}
1b: & \quad P_{d}(i_1) = \prod_{i=1}^{n} p_{b_i} (1 - p_{b_i}) \\
2b: & \quad P_{d}(i_2, i_1) = \prod_{i=1}^{n} p_{b_i} (1 - p_{b_i})(1 - p_{b_i}) \\
\vdots \\
\end{align*} \]

After deriving the bit-error probabilities \(P_{e_n}\) and \(P_{d_n}\) caused by noise and misregistration separately, we will consider the combined error probability caused by both noise contamination and frame misalignment.

3) Bit Error Due to the Combined Effect of Noise and Misregistration: A pixel value waiting to be evaluated by the similarity criterion can fall into one of two categories: 1) a correct one, unaffected by either noise or misalignment, or affected by both noise and misalignment, 2) an erroneous one, affected by either noise or misalignment. Any single inversion of bit value due to one and only one of these two factors will cause an error. The analysis proceeds by assuming that \(n\) bits, \((b_{i_n}, \cdots, b_{i_1}) = b\), are altered due to the combined effect of noise and displacement where \(0 \leq n \leq 8\). Also, there are \(m\) bits caused by misregistration where \(0 \leq m \leq 8\). These \(m\) bits can be further divided into two classes based on whether their values will be altered again by the noise.

Among these \(m\) bits, \(a\) bits will not be affected by the noise and remained incorrect, i.e.,
\[ a = (b_{i_k+1}, \cdots, b_{i_{k+1}}) \in b \]

where \(1 \leq a \leq \min(n, m)\). The remaining \((m - a)\) bits alter their values again due to noise corruption and the final values are correct, i.e., the set of bits \(c = (c_{i_{n-a}}, \cdots, c_{i_1})\) suffers double inversion and remains intact. The probability of \(m\) bit errors caused by misregistration is \(P_{d_m}(c_{i_{n-a}}, \cdots, c_{i_1}, b_{i_{k+1}}, \cdots, b_{i_{k+1}})\), or in more compact form, \(P_{d_m} (c, a)\).

Given these observations, in order to change exactly \(n\) bits, the noise must affect both \(c\) and \(b - a\), and the probability of the above event is \(P_{e_{n-a}} (c, b - a)\). So, the probability of \(n\) bit errors, \((b_{i_n}, \cdots, b_{i_1})\) due to the combined effects of misregistration and noise is
\[ P_{e_n}(i_n, \cdots, i_1) = \sum_{a=0}^{n} \sum_{b=0}^{n-a} P_{e_{n-a}}(c, b - a) \]
where \(2^n\) represents all the subsets of \(b\).

When we use the CBC method to estimate the displacement between two frames, \(I_1\) and \(I_2\), a mismatch occurs when the bit values corresponding to the same spatial location are not the same. This happens when the corresponding bit values in the
reference frame $I_1$ and registration frame $I_2$ are inconsistent. This bit value inconsistency can be caused by a bit error in either frame but not both. The reference frame $I_1$ is affected by the noise only, while the registration frame $I_2$ is affected by both noise and misregistration. In what follows, we will derive the probability that there are $n$ mismatch bits between frames $I_1$ and $I_2$.

4) Mismatch Bits Between Frames: Assume $n$ mismatch bits, $(b_{i_1}, \ldots, b_{i_n}) = b$, exist between $I_1$ and $I_2$ where $0 \leq n \leq 8$. We proceed first by further assuming there are $m$ bit errors caused by the noise in $I_1$ where $0 \leq m \leq 8$. These $m$ bits can be divided into two classes based on whether their values will match the corresponding bit values in $I_2$. Bits that are erroneous in both frames will remain matching with each other, while any error bits existing in only one of two frames will cause a mismatch.

Among these $m$ bits, $a$ bits will not have the corresponding error bits in $I_2$, thus remaining mismatched, i.e.,

$$a = (b_{i_{a_1}}, \ldots, b_{i_{a_a}}) \in b$$

where $0 \leq a \leq \min(n, m)$. The remaining $(m - a)$ bits find the matching error bits in $I_2$ and the comparison result is a match, i.e.,

$$c = (c_{n_{b_1}}, \ldots, c_{n_{b_m}}).$$

The probability of having $m$ bit errors in the reference frame is $P_{m_n}(c, a)$.

Given the above observations, in order to have exactly $n$ mismatch bits, the combined effects of noise and displacement must affect both $c$ and $b - a = (b_{i_1}, \ldots, b_{i_{k+a}}, b_{i_{k+a+1}}, \ldots, b_{i_{k+m}})$ in frame $I_2$. The probability of the above event is $P_{m_n}(c, a)$ and $P_{m_n}(b - a)$ in frame $I_1$ and $I_2$ are independent. So, the probability of $n$ mismatch bits, $(b_{i_1}, \ldots, b_{i_n})$, between two misaligned frames $I_1$ and $I_2$ is

$$P_{m_n}(i_1, \ldots, i_n) = \sum_{a=0}^{m} \sum_{b=0}^{m} P_{m_n}(c, a) \cdot P_{m_n}(b - a),$$

and the probability of $n$ mismatch bits is just a summation of all possible combinations of these $n$ mismatch bits:

$$P_{m_n} = \sum_{i=n}^{m} \sum_{i_1=1}^{i-1} \sum_{i_2=1}^{i_1} \cdots \sum_{i_{n-1}=1}^{i_{n-2}} P_{m_n}(i_1, \ldots, i_n).$$

The average number of matching bits in a byte when two frames are misaligned can be found to be equal to

$$8 - \sum_{i=1}^{8} iP_{m_n}.$$
the aforementioned angiographic sequence. The superiority of 
CBC over the SSC method is demonstrated by showing that 
the correct registration result is reached by CBC and missed by 
the SSC method. Finally, the effect of using different number 
of comparison bits in the CBC criterion is studied.

A sequence of low-contrast coronary angiographic images 

is used to demonstrate the performance of the registration 
algorithm. A typical 512 x 512 pixel frame from this sequence 

is shown in Fig. 2. A 50 x 100 pixel area of interest in the 

upper left corner, which includes a section of coronary arteries, 
is taken as input for the registration algorithm.

A. CBC versus SSC

First, the CBC method is used to find the translational 
displacement between two succeeding digital angiograms. 

Fig. 3 shows the registration value in the neighborhood of 
the correct displacement vector (0, 10). In this case, the 
CBC method successfully reaches the maximum value at 
the correct registration position. Fig. 4 shows the result of 
applying the SSC criterion on the same pair of coronary 
images. The undesirable fluctuation of the number of sign 
change corresponding to different displacement vectors is an 
indication of inappropriate usage of a Gaussian noise model 
in the description of low-intensity images. As discussed in 
previous sections, quantum noise predominates in this case. 
A signal-dependent Poisson model is a better candidate to 
describe the noise distribution. However, the presence of this 
discrete, unsymmetrical noise leads to the instability of SSC 
method. The CBC method can be seen to perform far better 
than SSC does in this environment.

Using the signal-dependent Poisson noise model described 
in (4), we add computer-generated noise to one frame chosen 
from the aforementioned sequence. The first synthetic noisy 
frame is obtained by setting the proportionality factor $\lambda$ equal 
to 0.75, and the second one by setting $\lambda$ equal to 0.77. 
Since there is no displacement between these two noise-
corrupted frames, the correct displacement vector is (0, 0). 
Fig. 5 shows the registration value using the CBC method in 
the neighborhood of (0, 0). The maximum value is reached
The reason for the failure of SSC method can be attributed to the unsymmetric noise distribution inherent in the subtraction image. In contrast, CBC method will not be affected by the distribution of noise model and is well-suited for medical image applications.

**B. Number of Comparison Bits**

The appropriate setting of the number of comparison bits is determined by the image characteristics. When the noise levels are high, the lower order bits easily get contaminated. Thus they are not reliable candidates for comparison. When the image contrast is low, the higher order bits tend to have the same bit values locally. Including them among the comparison bits will flatten the search surface, thereby lowering the noise tolerance. In addition, using only those bits that yield displacement information reduces the computational cost of the implementation.

The same pair of images described previously is used to study the effect of applying different numbers of comparison bits in the CBC method. Fig. 7 shows the registration value at the correct registration vector. Fig. 6 shows the registration value using the SSC method in the same area. The peak value is obtained at (1, 0) instead of (0, 0). Misregistration occurs when the SSC method is used.
found by choosing the middle two bits for comparison in the neighborhood of the correct displacement vector (0, 10). Fig. 8 shows the case of using eight comparison bits. Peak values are reached in both settings irrespective of the number of chosen comparison bits. This study shows the robustness of using CBC method for various settings, and in each case studied, correct registration is always reached.

To demonstrate the effect of contrast level, an iterative histogram peak sharpening technique is performed on the same pair of angiograms to obtain sharper images [13]. The histograms before and after the sharpening operation are shown in Fig. 9. The CBC method (Fig. 10) using eight comparison bits is applied and the registration value is of bits used for each pixel chosen for comparison. An improved performance at a reduced computational cost can thus be reached.

Finally, the registration algorithm incorporating this criterion renders computational efficiency and can be implemented as a neural network. The feasibility of using a coherent neural architecture to implement both image registration and multichannel image restoration algorithms shows promising potential for single-chip real-time applications.

REFERENCES