Resolution Enhancement Using EM Algorithm
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Abstract

The EM (expectation-maximization) algorithm is a broadly applicable method for calculating maximum likelihood estimates given incomplete data [1]. EM algorithms have received considerable attention due to their computation feasibility in tomographic image reconstruction [2–4], symbol detection [5] and parameter estimation [6]. However, it is less recognized that EM algorithms can be equally applicable to image enhancement applications encountered in scanning, reproduction and rendering processes. No past techniques surveyed can incorporate the potentially complex nature of various image formation processes into a simple probability density array as EM procedure does. In this paper, a resolution enhancement method utilizing EM procedure is proposed. By dynamically giving a priori probability distribution suited for a specific application environment currently considered, the proposed method provides a general framework for rendering good image quality at the designated resolution for a large class of image formation process.

In the EM algorithm, a postulated, unobservable “complete data set” is employed to facilitate the process of maximizing the likelihood function of the measured data. The actual calculation consists of a sequence of alternating expectation steps (the E-step, in which the conditional expectation of a new likelihood function defined on the complete data set is derived) and maximization steps (the M-step, where a set of new estimates is obtained by maximizing the conditional expectation formulated in the E-step). This iterative process has the desirable properties of maximizing the likelihood function defined on the measured data monotonically, and converging to a global maximum at a unique point [7].

A schematic representation of the application of the EM algorithm to resolution enhancement encountered in the scanning process is showed in Figure 1. A fluorescent light source pulled by a tracking mechanism from top to bottom emits light on the surface of the scanned hardcopy. The intensity of the light reflected is then detected by an array of light sensors. However, the intensity registered by any specific sensor is affected not only by the area bounded by scanner resolution, but also neighboring areas due to the diffusion of light. The proposed method utilizing EM procedure can be used to compensate the above effect, restore the original density, or increase resolution if desired.

![Figure 1. Image formation process and the spatial relationship between sensors and image cells.](image-url)

Let the intensities detected by the sensor array correspond to the incomplete data set \( \{S_{ij}\} \), and the complete data set \( \{I_{ij,kl}\} \), defined as the physically unmeasurable actual image intensity located at cell \((k, l)\), distributed by parameter yet to be determined \( \lambda_{kl} \), and registered in sensor \((i, j)\). The dimensions of \( S \) and \( \lambda \) need not to be the same. Due to the cost of the sensor, the number of sensors is almost always less than the number of image cells in the real
application. Hence, the problems we are targeting can be stated as: “*Given the limited spatial resolution of sensor arrays, how can we best compensate the physical configuration of the underlying imaging digitization system and render the desired resolution of image cells?*” No past techniques surveyed can incorporate the potentially complex nature of image formation process into a simple probability array as EM procedure does. The new enhancement method proposed in this paper can obtain resolution at any desired scale given the intensities registered by the finite number of sensors, and the spatial correspondence between sensors \((i, j)\) and image cells \((k, l)\). The EM operation for a one-to-one mapping between \(S\) and \(I\) represents image density compensation or recalibration, while a many-to-one mapping means resolution enhancement. By incorporating all physical factors involved in the image formation process, the probability density function \(\{ \text{\(C_{ij, kl}\)}\}\) representing the light intensity emitted from cell \((k, l)\) and detected by sensor \((i, j)\) can be formed. An iterative equation formulated by employing the EM paradigm is as follows:

\[
A_{kl}^{(n+1)} = \left( \sum_{i} \sum_{j} C_{ij, kl} A_{ij}^{(n)} \right) \left( \sum_{i} \sum_{j} C_{ij, kl} S_{ij} \right) \sum_{m} \sum_{n} C_{ij, mn} A_{mn}^{(n)}
\]

The same principle can be adapted to various imaging systems by incorporating all physical factors into \(\{ \text{\(C_{ij, kl}\)}\}\) and specifying different spatial relationship between cells and sensors. Take a rod-like arrangement of sensors for example, the \(\{ \text{\(C_{ij, kl}\)}\}\) can be confined to a rectangular area, served as an indication for the line sensor setup, with amplitude inverse proportional to the distance from the center cell considered reflecting an energy drop (ref. Figure 2). If a one-to-one spatial mapping between image cell and sensor is specified, then the EM procedure redistribute the intensity of image cell detected according to the probability density matrix \(\{ \text{\(C_{ij, kl}\)}\}\) postulated. On the other hand, if a many-to-one mapping is specified, an interpolation of cells is performed based on the cells detected and the probability density matrix \(\{ \text{\(C_{ij, kl}\)}\}\). An enhancement in resolution is achieved. In comparison with other interpolation techniques, e.g. linear interpolation between neighboring image cells, the EM algorithms has the advantage of incorporating all the physical factors affecting a complex image formation into a single probability density matrix \(\{ \text{\(C_{ij, kl}\)}\}\) and thus gives better results.

![Figure 2. The probability density matrix \(\{ \text{\(C_{ij, kl}\)}\}\) for an exemplary rod-like arrangement of sensors](image)

The preliminary computer simulation performed on a natural tone gray scale image shows increasing contrast after the EM recalibration procedure (*Regrettably, due to the time required to produce the image hardcopy, we are unable to include the images mentioned in this abstract before the deadline, as a demonstration of the power of this proposed algorithm.*). The resolution enhancement obtained on a 3-to-1 scale also compares favorably with that of linear interpolation or simple duplication. A substantial amount of high frequency component is included after the enhancement process in comparison with other techniques. The major contribution of this paper is proposing a general framework utilizing EM algorithm on the image enhancement of a complex, not necessarily well-defined image formation process. The results show promising improvement on image quality that deserves
further investigation.

References


